International Youth Mathematics Contest 2007

Hongkong Elementary Mathematics International Contest (HEMIC)

Individual Competition

Time allowed : 1 hour 30 minutes

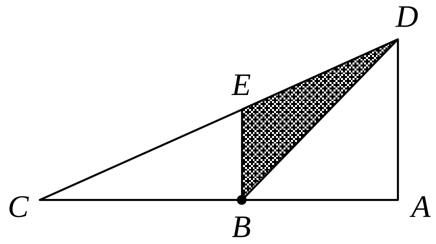
Hong Kong : 29 July – 2 August 2007

Instructions:

- Write down your name, team name and candidate number on the answer sheet.
- Write down all answers on the answer sheet.
- Answer all 15 problems. Problems are in ascending order of level of difficulty. Only NUMERICAL answers are needed.
- Each problem is worth 6 points and the total is 90 points.
- For problems involving more than one answer, points are given only when ALL answers are correct.
- Take π = 3.14 if necessary.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All materials will be collected at the end of the competition.

1. The product of two three-digit numbers $\overline{abc}$ and $\overline{cba}$ is 396396, where a > c. Find the value of $\overline{abc}$.

2. In a right-angled triangle *ACD*, the area of shaded region is 10 cm^2, as shown in the figure below. *AD* = 5 cm, *AB = BC, DE = EC.* Find the length of *AB*.



3. A wooden rectangular block, 4 cm × 5 cm × 6 cm, is painted red and then cut into several 1 cm × 1 cm × 1 cm cubes. What is the ratio of the number of cubes with two red faces to the number of cubes with three red faces?

4. Eve said to her mother, "If I reverse the two-digits of my age, I will get your age." Her mother said, "Tomorrow is my birthday, and my age will then be twice your age." It is known that their birthdays are not on the same day. How old is Eve?

5. Find how many three-digit numbers satisfy all the following conditions:
if it is divided by 2, the remainder is 1,
if it is divided by 3, the remainder is 2,
if it is divided by 4, the remainder is 3,
if it is divided by 5, the remainder is 4,
if it is divided by 8, the remainder is 7.

6. A giraffe lives in an area shaped in the form of a right-angled triangle. The base and the height of the triangle are 12 m and 16 m respectively. The area is surrounded by a fence. The giraffe can eat the grass outside the fence at a maximum distance of 2 m. What is the maximum area outside the fence, in which the grass can be eaten by the giraffe?

7. Mary and Peter are running around a circular track of 400 m. Mary's speed equals $\frac{3}{5}$ of Peter's. They start running at the same point and the same time, but in opposite directions. 200 seconds later, they have met four times. How many metres per second does Peter run faster than Mary?

8. Evaluate $2^{2007}-\left(2^{2006}+2^{2005}+2^{2004}+......+2^{3}+2^{2}+2+1\right)$

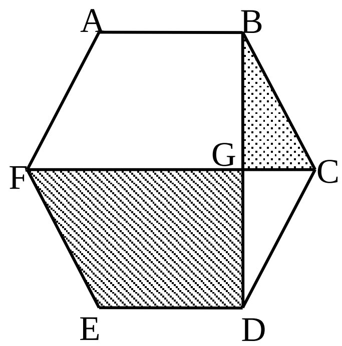
9. A, B and C are stamp-collectors. A has 18 stamps more than B. The ratio of the number of stamps of B to that of C is 7:5. The ratio of the sum of B's and C's stamps to that of A's is 6:5. How many stamps does C have?

10. What is the smallest amount of numbers in the product $1\times2\times3\times4\times...\times26\times27$ that should be removed so that the product of the remaining numbers is a perfect square?

11. Train A and Train B travel towards each other from Town A and Town B respectively, at a constant speed. The two towns are 1320 kilometers apart. After the two trains meet, Train A takes 5 hours to reach Town B while Train B takes 7.2 hours to reach Town A. How many kilometers does Train A run per hour?

12. Balls of the same size and weight are placed in a container. There are 8 different colors and 90 balls in each color. What is the minimum number of balls that must be drawn from the container in order to get balls of 4 different colors with at least 9 balls for each color?

13. In a regular hexagon *ABCDEF*, two diagonals, *FC* and *BD*, intersect at *G*. What is the ratio of the area of Δ*BCG* to that of quadrilateral *FEDG?*



14. There are three prime numbers. If the sum of their squares is 5070, what is the product of these three numbers?

15. Let *ABCDEF* be a regular hexagon. *O* is the centre of the hexagon. *M* and *N* are the mid-points of *DE* and *OB* respectively. If the sum of areas of ΔFNO and ΔFME is 3 cm^2, find the area of the hexagon.

~ End of Paper ~