Taiwan International
Mathematics Competition 2012
(TAIMC 2012)

World Conference on the Mathematically Gifted Students
---- the Role of Educators and Parents
Taipei, Taiwan, 23rd~28th July 2012

Elementary Mathematics International Contest

Individual Contest

Time limit: 90 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Write down your name, your contestant number and your team's name on the answer sheet.
- Write down all answers on the answer sheet. Only Arabic NUMERICAL answers are needed.
- Answer all 15 problems. Each problem is worth 10 points and the total is 150 points. For problems involving more than one answer, full credit will be given only if ALL answers are correct, no partial credit will be given. There is no penalty for a wrong answer.
- Diagrams shown may not be drawn to scale.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All papers shall be collected at the end of this test.

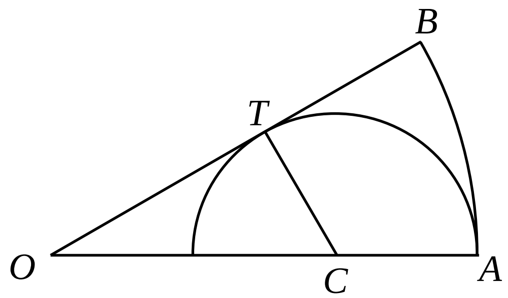
English Version

1. In how many ways can 20 identical pencils be distributed among three girls so that each gets at least 1 pencil?

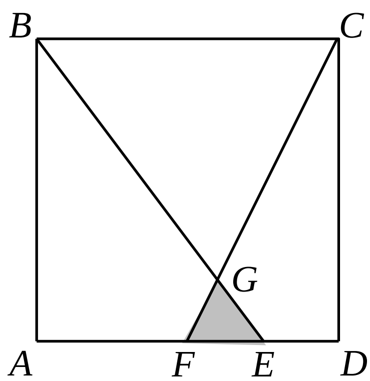
2. On a circular highway, one has to pay toll charges at three places. In clockwise order, they are a bridge which costs \$1 to cross, a tunnel which costs \$3 to pass through, and the dam of a reservoir which costs \$5 to go on top. Starting on the highway between the dam and the bridge, a car goes clockwise and pays toll-charges until the total bill amounts to \$130. How much does it have to pay at the next place if he continues?

3. When a two-digit number is increased by 4, the sum of its digits is equal to half of the sum of the digits of the original number. How many possible values are there for such a two-digit number?

4. In the diagram below, OAB is a circular sector with $OA = OB$ and $\angle AOB = 30°$. A semicircle passing through A is drawn with centre C on OA, touching OB at some point T. What is the ratio of the area of the semicircle to the area of the circular sector OAB?

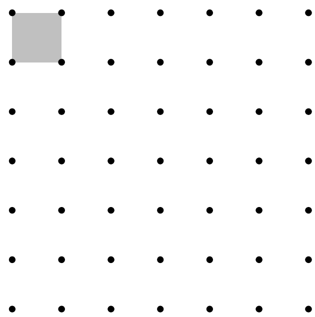


5. $ABCD$ is a square with total area 36 cm^2. F is the midpoint of AD and E is the midpoint of FD. BE and CF intersect at G. What is the area, in cm^2, of triangle EFG?



6. In a village, friendship among girls is mutual. Each girl has either exactly one friend or exactly two friends among themselves. One morning, all girls with two friends wear red hats and the other girls all wear blue hats. It turns out that any two friends wear hats of different colours. In the afternoon, 10 girls change their red hats into blue hats and 12 girls change their blue hats into red hats. Now it turns out that any two friends wear hats of the same colour. How many girls are there in the village? (A girl can only change her hat once.)

7. The diagram below shows a 7×7 grid in which the area of each unit cell (one of which is shaded) is 1 cm^2. Four congruent squares are drawn on this grid. The vertices of each square are chosen among the 49 dots, and two squares may not have any point in common. What is the maximum area, in cm^2, of one of these four squares?



8. The sum of 1006 different positive integers is 1019057. If none of them is greater than 2012, what is the minimum number of these integers which must be odd?

9. The desks in the TAIMC contest room are arranged in a 6×6 configuration. Two contestants are neighbours if they occupy adjacent seats along a row, a column or a diagonal. Thus a contestant in a seat at a corner of the room has 3 neighbours, a contestant in a seat on an edge of the room has 5 neighbours, and a contestant in a seat in the interior of the room has 8 neighbours. After the contest, a contestant gets a prize if at most one neighbour has a score greater than or equal to the score of the contestant. What is maximum number of prize-winners?

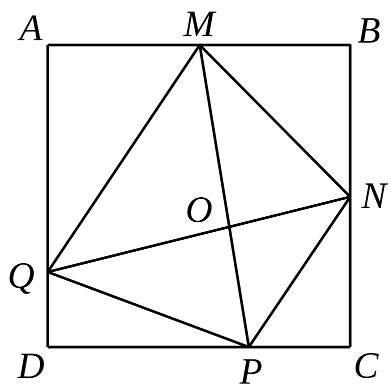
10. The sum of two positive integers is 7 times their difference. The product of the same two numbers is 36 times their difference. What is the larger one of these two numbers?

11. In a competition, every student from school A and from school B is a gold medalist, a silver medalist or a bronze medalist. The number of gold medalist from each school is the same. The ratio of the percentage of students who are gold medalist from school A to that from school B is 5:6. The ratio of the number of silver medalists from school A to that from school B is 9:2. The percentage of students who are silver medalists from both school is 20%. If 50% of the students from school A are bronze medalists, what percentage of the students from school B are gold medalists?

12. We start with the fraction $\frac{5}{6}$. In each move, we can either increase the numerator by 6 or increases the denominator by 5, but not both. What is the minimum number of moves to make the value of the fraction equal to $\frac{5}{6}$ again?

13. Five consecutive two-digit numbers are such that 37 is a divisor of the sum of three of them, and 71 is also a divisor of the sum of three of them. What is the largest of these five numbers?

14. *ABCD* is a square. *M* is the midpoint of *AB* and *N* is the midpoint of *BC*. *P* is a point on *CD* such that *CP* = 4 cm and *PD* = 8 cm, *Q* is a point on *DA* such that *DQ* = 3 cm. *O* is the point of intersection of *MP* and *NQ*. Compare the areas of the two triangles in each of the pairs (*QOM*, *QAM*), (*MON*, *MBN*), (*NOP*, *NCP*) and (*POQ*, *PDQ*). In cm^2, what is the maximum value of these four differences?



15. Right before Carol was born, the age of Eric is equal to the sum of the ages of Alice, Ben and Debra, and the average age of the four was 19. In 2010, the age of Debra was 8 more than the sum of the ages of Ben and Carol, and the average age of the five was 35.2. In 2012, the average age of Ben, Carol, Debra and Eric is 39.5. What is the age of Ben in 2012?