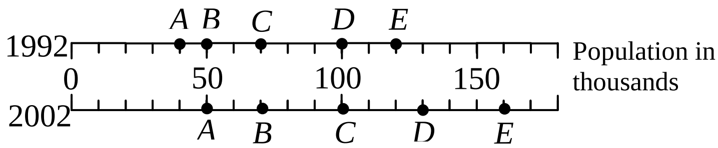
Invitational World Youth Mathematics Intercity Competition 2002

Individual Contest

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. On each horizontal line in the figure below, the five large dots indicate the populations of five branches of City Montessori School in Lucknow: *A, B, C, D* and *E* in the year indicated. Which City Montessori School, Lucknow had the greatest percentage increase in population from 1992 to 2002?

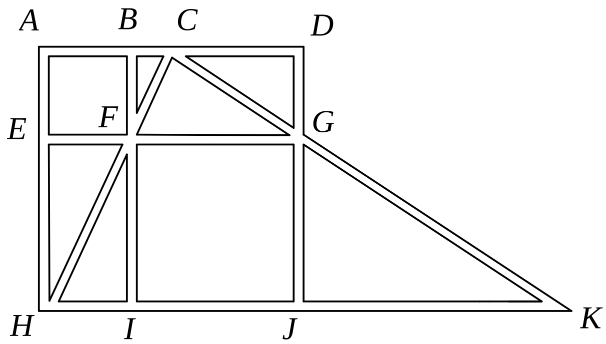


2. If $x = \dfrac{\sqrt{(a+2b)}+\sqrt{(a-2b)}}{\sqrt{(a+2b)}-\sqrt{(a-2b)}}$, what is the numerical value of $bx^2 - ax + b$?

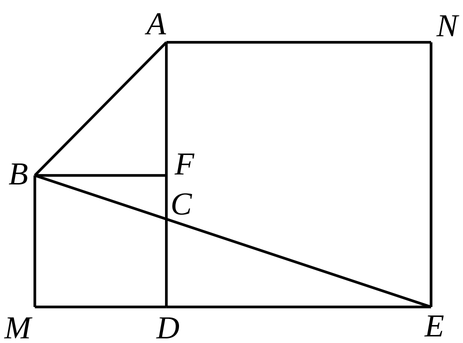
3. To find the value of x^8 given x, you need three arithmetic operations: $x^2 = x \cdot x$, $x^4 = x^2 \cdot x^2$ and $x^8 = x^4 \cdot x^4$. To find x^{15}, five operations will do: the first three of them are the same; then $x^{16} = x^8 \cdot x^8$ and $x^{15} = x^{16} \div x$. What is the minimum number of operations (multiplications and divisions) will be needed to find the value of x^{1000}?

4. Let $P(x) = x^4 + ax^3 + bx^2 + cx + d$ where a, b, c and d are constants. If $P(1) = 10$, $P(2) = 20$, $P(3) = 30$, what is the value of $P(10) + P(-6)$?

5. The diagram below shows the street map of a city. If three police offcers are to be positioned at street corners so that any point on any street can be seen by at least one offcer, what are the letter codes of these street corners?



6. *ADEN* is a square. *BMDF* is a square such that *F* lies on *AD* and *M* lies on the extension of *ED*. *C* is the point of intersection of *AD* and *BE*. If the area of triangle *CDE* is 6 square units, what is the area of triangle *ABC*?



7. If the 18-digit number A36 405 489 812 706 44B is divisible by 99, what are all the possible values of (A, B)?

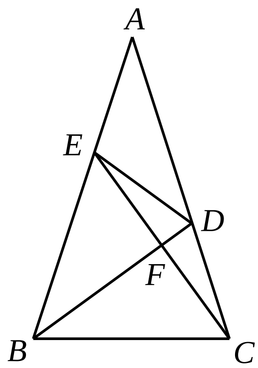
8. Ten people stand in a line. The first goes to the back of the line and the next person sits down so that the person who was third in the line is now first in line. Now the person on the first in line goes to the back of the line and the next person sits down. This process is repeated until only one person remains. What was the original position in line of the only remaining person?

9. In triangle ABC, bisectors AA_1, BB_1 and CC_1 of the interior angles are drawn. If $\angle ABC = 120°$, what is the measure of $\angle A_1B_1C_1$ =?

10. For how many different real values of k do there exist real numbers x, y and z such that $\frac{x+y}{z} = \frac{y+z}{x} = \frac{z+x}{y} = k$?

11. L is a point on the diagonal AC of a square $ABCD$ such that AL = 3 LC. K is the midpoint of AB. What is the measure of $\angle KLD$?

12. In triangle ABC, $\angle A = 36°$, $\angle ACB = 72°$. D is a point on AC such that BD bisects $\angle ABC$. E is a point on AB such that CE is perpendicular to BD. How many isosceles triangles are in figure?



Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. There are two distinct 2-digit numbers which have the same units digit but different tens digits. The quotient when one of them is divided by 9 is equal to the remainder when the other is divided by 9, and vice versa. What is the common units digit?

2. Solve for x, y and z if

$$(x+y)(x+z) = 15$$
$$(y+z)(y+x) = 18$$
$$(z+x)(z+y) = 30$$

3. In triangle ABC, D is the point on BC such that AD bisects $\angle CAB$, and M is the midpoint of BC. E is the point on the extension of BA such that ME is parallel to AD and intersects AC at F. Prove that $\overline{BE} = \overline{CF} = \frac{1}{2}(\overline{AB} + \overline{AC})$.